PROPERTIES OF ANALOGUE AND DIGITAL SIGNALS

1. ANALOGUE AUDIO SIGNALS

Sound, or technically speaking, audio signal, is a time dependent variation of air pressure. Sound is an ANALOGUE signal.

Electronic processing of sound requires a CONVERSION of variation of air pressure to a variation of an electrical signal, normally a voltage or a current. The microphone is used as "air-pressure to voltage converter", while the loud-speaker is the "voltage to air-pressure converter". Microphone and loud-speaker are analogue converters: The relationship between voltage and air-pressure is linear (in the ideal case).

The full definition of an ANALOGUE SIGNAL:
1. It is defined for ANY instant of time.
2. It may have an INFINITE number of different instantaneous values.

Audio signals are mainly PERIODICAL signals. This means they repeat themselves many times. But the shape of the curves is very irregular and can not easily be represented in a mathematical form.

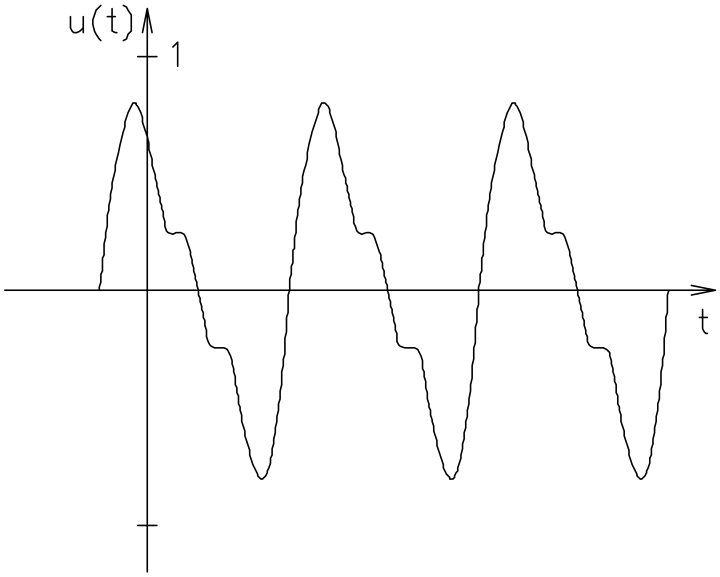


Fig. 1.1.1.
The normal representation of an analog, time-varying signal. The fact that it is represented by a continuous line shows, that it is defined for any instant and that it may have an infinite number of different values.

The periodical electrical signals can be made visible by means of an oscilloscope.

2. THE FOURIER ANALYSIS

J.P. de Fourier (French mathematician) has shown already in 1822 that ANY PERIODICAL signal can be represented by a SUM OF SINUSOIDAL FUNCTIONS.

The full understanding of the Fourier analysis requires a sound knowledge of advanced mathematics, but the basic ideas of it can be understood by some practical considerations.

The periodical, time-varying signal s(t) can be represented by

$$s(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n \cdot 2\pi f t + \Phi_n)$$

Where

n	is an integer,
A_o	is the average or d.c. value of the signal,
A_1	is the amplitude of the FUNDAMENTAL frequency f_1,
A_2 to A_∞	are the amplitudes of the HARMONICS, which have frequencies, which are always a multiple of the fundamental ($n \times f_1$).

The consideration of the phase Φ_n can be omitted in most cases of analysing audio signals.

The Fourier analysis has two important practical conclusions:

1. A periodical function can now be described in mathematical form, because the sine-function is clearly defined.

2. The human ear does not follow the time function of the audio signal, but analyses the frequencies contained in it.
   The ear performs a "Fourier analysis" of the signal.

As a conclusion of the Fourier analysis, any periodical signal can be represented in the TIME DOMAIN, this means with a horizontal time axis, or in the FREQUENCY DOMAIN, with a horizontal frequency axis.

Changing the representation of a signal form the time domain to the frequency domain is called a FOURIER TRANSFORM.

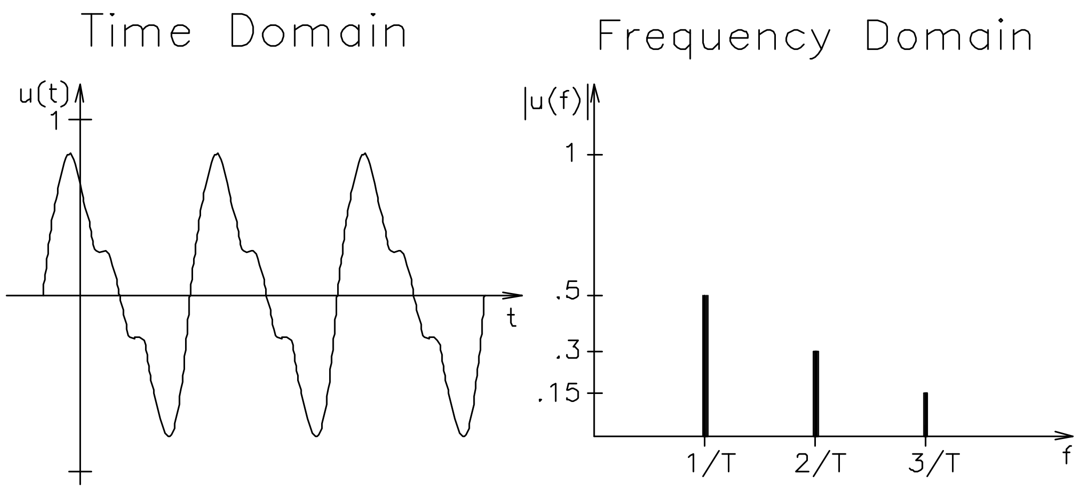


Fig. 1.2.1.
Comparison of the different representation of the same periodical signal in the time and in the frequency domain.

We can select among these two representations freely the one which is more convenient to describe a signal for a certain purpose. The representation in the frequency domain plays a big roll in the discussion of digital signal processing.

While the oscilloscope is the suitable device to represent periodical functions in the time domain, a FREQUENCY ANALYSER can represent the signal in the frequency domain.

Audio signals cover frequencies between 40Hz and 15Khz and ANY frequency in this range may occur. Any of these frequencies can have a level between 0 and 100%. When the whole possible audio range is to be represented in the frequency domain, it will cover a certain AREA in the amplitude/frequency-response.

Fig. 1.2.2.
Representation of the entire audio range between 40Hz and 15kHz in the frequency domain.

3. DIGITAL SIGNAL

The Latin word "digit" means "finger", standing for a simple aid for counting. Counting by using fingers allows a limited number of figures only.

The digital technique uses two figures only: 1 and 0. Therefore it uses BINARY DIGITAL SIGNALS. The binary digits are called BITs.

When we want to count binary to more than 1, we have to use more digits. Using two digits allows to count to 2^2-1 = 3 and using 3 allows two count to 2^3-1 = 7. So, by extending the number of digits, we can count up to any number.

The position of each digits defines its value:

$$x = b_n \times 2^n + b_{n-1} \times 2^{n-1} + ... + b_1 \times 2^1 + b_0 \times 2^0$$

(b is the bit, having values of 0 or 1)

Although very high values can be represented, the number of DIFFERENT values is limited. Between the neighbouring values 1001101_B and 1001110_B there is no other value. The step between the two numbers is 1. We say the digital system has DISCRETE VALUES.

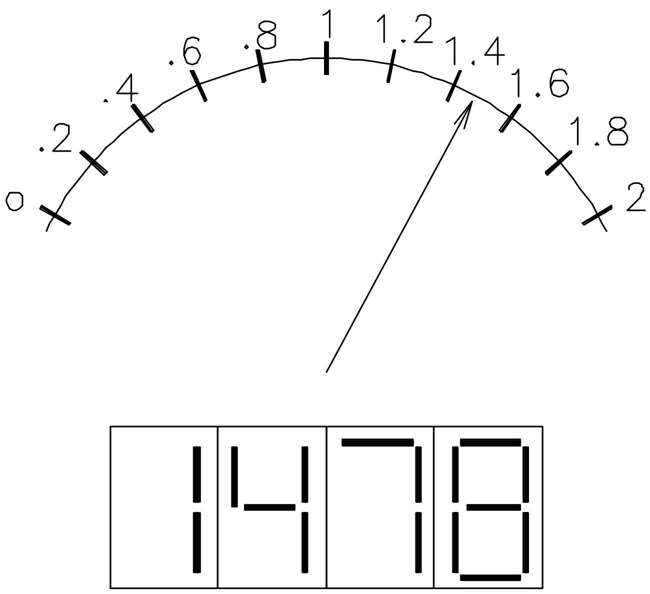


Fig. 1.3.1.
Comparing the dial of an analogue meter and the display of digital meter:
The analogue meter may have an infinite number of different pointer indications (although we might not be able to distinguish them).

The digital display can only indicate numbers between 0000 and 1999. This allows to display only 2000 different values. Still we consider the digital display to be more precise than the analogue one.

When an electrical signal is represented by binary numbers, only these voltages are allowed, which can be represented by these numbers. This means, voltages, which have values which lie between two numbers, are not defined and are not possible.
We say only DISCRETE VALUES are allowed.

As electrical signals always require a certain time, to change from one value to another, there will be times, when the signal is not defined. So the signal can only be represented at certain, DISCRETE TIMES.

The full definition of a digital signal is:
1. It is defined only for a limited number of values.
2. It is defined only for certain instances.

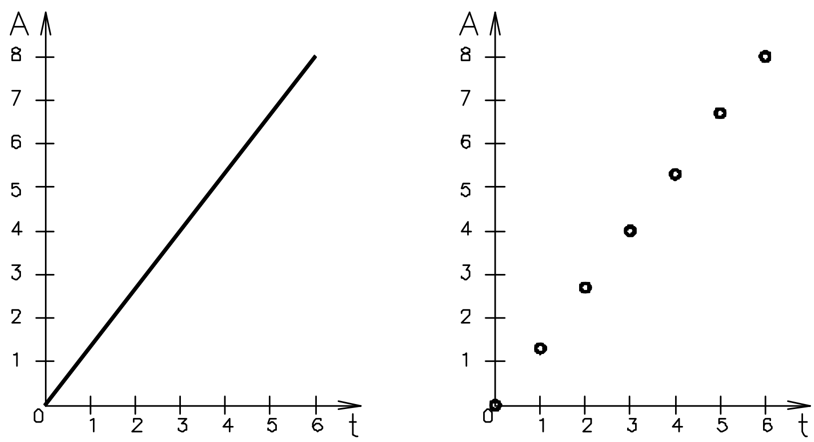


Fig. 1.3.2.

A simple analogue signal:
A constantly rising voltage.
The voltage is defined for any time of the shown interval and has an infinite number of different values.

A digital signal:
The digital value of the function is incremented by a constant value at each time. Between the intervals the function is not defined.

To convert an analogue signal to a digital signal, two operations are required:

1. The analogue signal must be evaluated at certain instances.

2. The detected analogue values must be converted to binary digital values.

For each of the two operations a certain device will be required.

Each evaluation and conversion of an analogue value, thus each DIGITAL SAMPLE of the analogue signal, will produce a certain amount of data. If a certain time, e.g. 1 second, of a analogue signal is to be converted to a digital signal, the amount of binary data produced depends on:

- The number of samples taken during the period.
- The number of bits used to represent one sample.

The total number of bits produced is just the product of samples and bits per sample.

Satisfactory conversion of audio signals to digital form requires to represent each sample by at least 12 bit and to take at least 30 000 samples per second. This produces a DATA STREAM of 360 000 bit/s.
This is a huge amount, which will fill one floppy disk within just 24 seconds.

The high data rate is one of the biggest problem in digital audio processing.
It requires high cost for data storage media, for data processing and for transmission equipment.

Therefore the application of digital audio became a big boost, when data reduction technologies were introduced, which reduce the data rate by the factor 4 to 12 and made digital audio broadcasting applicable.

4. SAMPLING

In order to convert an analogue to a digital signal, samples must be taken of the analogue signal.

Theoretically we could select the instances for taking samples of the analogue signal deliberately. But in order to get simple synchronization between the A/D and D/A conversion, it is convenient to use constant sample intervals, resulting in a constant SAMPLING FREQUENCY.

The sampling must be done in a very short time, to minimize the errors, which will be produced, if the analogue signal changes during the sampling process. As a result, the sampling will be controlled by short periodical pulses.

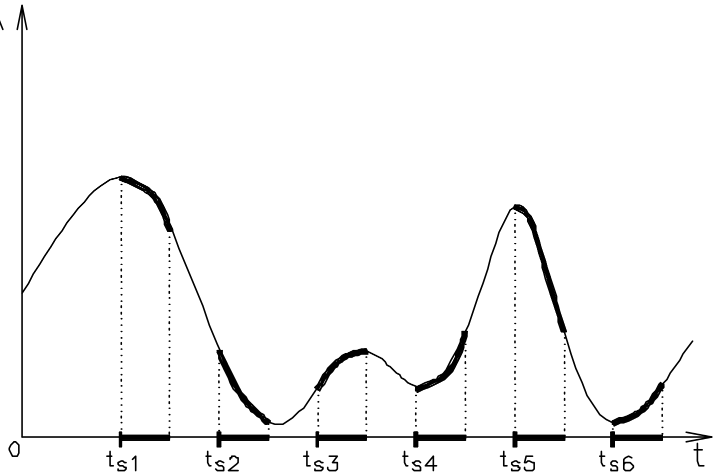


Fig. 1.4.1.
Sampling of an audio signal with periodical sample pulses. The pulses must be short to minimize the change of the sample-value during the sampling process.

With each sample more data are produced. In order to keep the data stream as low as possible, it is desirable to KEEP THE SAMPLING FREQUENCY AS LOW AS POSSIBLE.

To consider which sampling frequency is appropriate, let us analyze the sampling of different audio frequencies. The sampling frequency should be 10kHz, which means one sample each 100µs. The audio signal should be a sine wave of 2kHz, 4Khz and 6kHz.

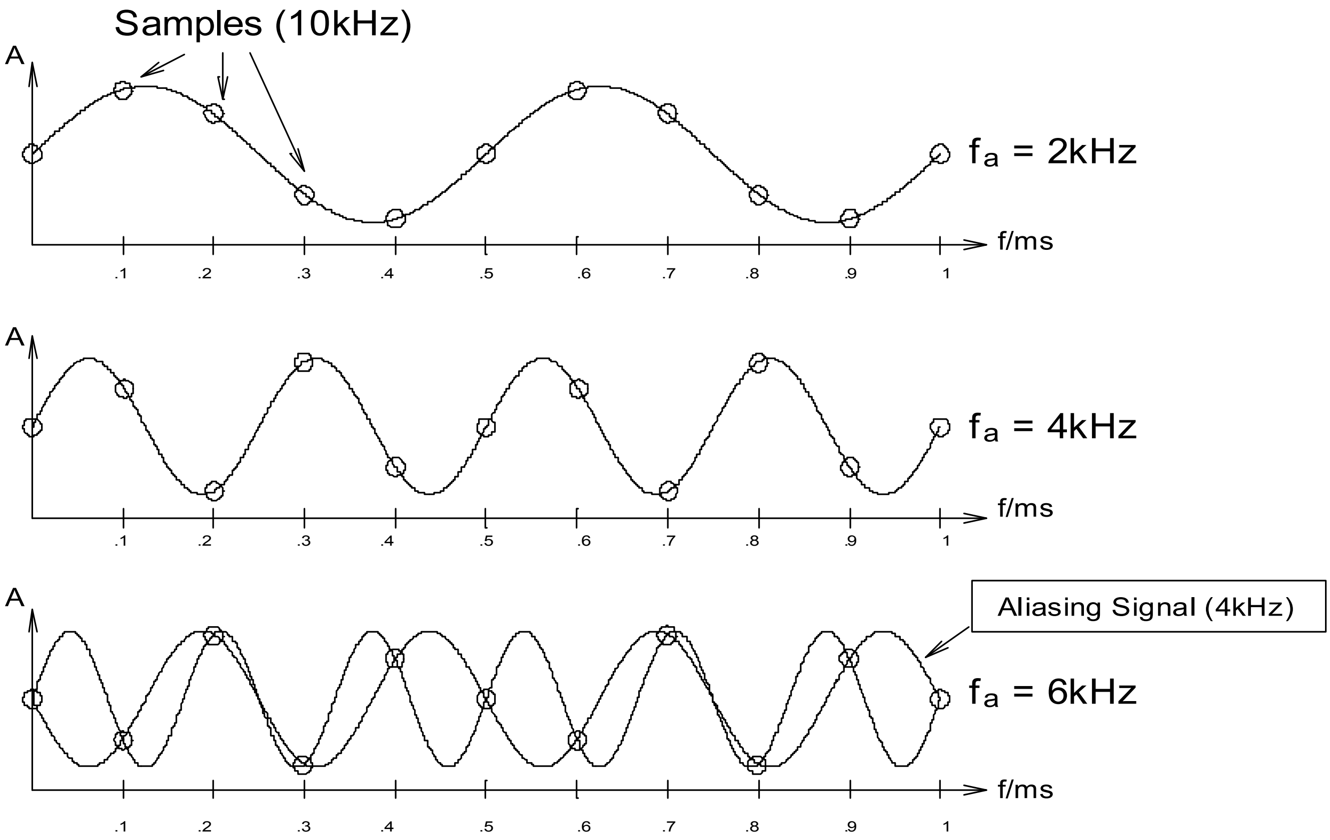


Fig. 1.4.3.
Representation of the sampling process in the time domain. Audio frequencies of 2kHz, 4kHz and 6kHz are sampled with 10kHz. It is to be seen, that at 6kHz, the samples will not represent the original frequency any more.

From the samples of the first two cases (2 and 4kHz), the original signal could be restored again, because the samples contain information about the amplitude and the frequency. In the last case the samples give an ambiguous result about the frequency of the signal.

Let us next consider the same situation in the frequency domain. To represent the sampling frequency, it is necessary to know, that the sampling process is a MULTIPLICATION of the sampling signal and the analogue signal. Multiplication of two signals of different frequencies is commonly referred to as AMPLITUDE MODULATION. So after sampling we will have the same frequency spectrum, which is known from am-signals.

Mathematically it can be described in the following way:

$$s(f) = A_a(f_a) + A_{ns}(n \cdot f_s) + A_{ns-}(n \cdot f_s - f_a) + A_{ns+}(n \cdot f_s + f_a)$$

(A_{ns} may be zero in case of suppressed carrier)

with:

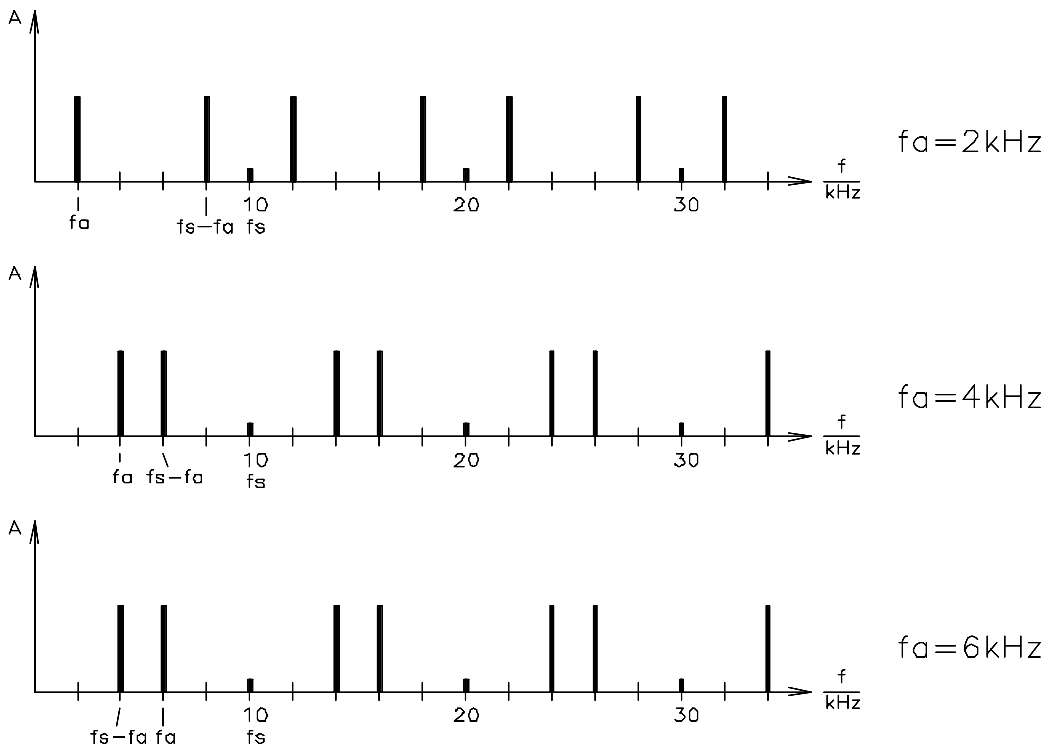
A_a The analogue signal,

A_{ns} The sampling signal with its harmonics (n=1 to ¥)

A_{ns+} The upper side band signal of the sampling signal,

A_{ns-} The lower side band signal of the sampling signal.

Fig. 1.4.4.



The same situation, as in the Fig. 1.4.3., but represented in the frequency domain.
By sampling the original analogue signal, a frequency spectrum is produced, which includes the sampling frequency and its harmonics. Each of these carry the analogue signal as lower and upper sideband.

The original signal can be retrieved from the spectrum by use of a low pass filter, which blocks all frequencies above the required signal frequency.

The representation in the frequency domain shows, that this works only for the case of the 2 and 4kHz. In case of the 6kHz there will be the spectral component A_{1s-} at $(1f_s\text{-}f_a)$ = 4khz. This frequency can not be separated form the original 6kHz any more. The original signal can not be retrieved.

The 4kHz in the above example is a frequency, which was originally not present in the analogue signal. It is an unwanted signal produced by the process of sampling. It is called an ALIASING SIGNAL.

Aliasing signals can only be avoided if
- **the analogue signal frequency is always less than half of the sampling frequency,**
  (Nyquist criterion)

or
- **the sampling frequency is more than twice the analogue frequency.**
  (Shannon criterion)

Both of these statements come out to the same result, but they produce different requirements to the system.

These relationships between the analogue signal and the sampling frequency were stated by Mr. SHANNON in 1948 and are now referred to as SHANNON's THEOREM:

To be able to retrieve the original analogue signal, the sampling frequency must be more than twice the highest frequency of the analogue signal.

If we apply this theorem to audio signals, we have to select the sampling frequency at least twice as high as the highest audio signal. As the upper end of audio range is not absolutely defined, it will make a difference whether we decide on 10kHz, 15kHz or 20kHz as highest frequency. The sampling frequency must then be higher than 20kHz or 30kHz or 40kHz respectively.

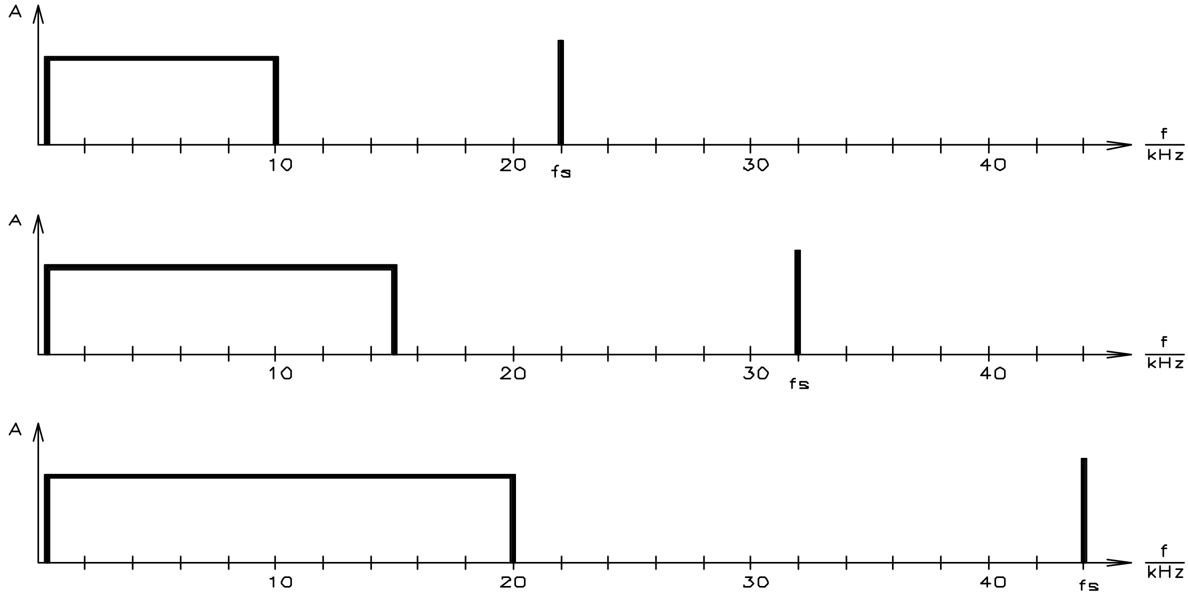


Fig. 1.4.5.
This figure shows, that the sampling frequency f_s *must be selected the higher, the wider the audio range is defined, to avoid aliasing.*

Unfortunately the industry did not succeed in standardizing the sampling frequency. Mainly three different frequencies are in use today:

32kHz used for transmission by the PTT and recommended by the European Broadcasting Union (EBU). This sampling frequency allows an audio band-width of up to 15kHz.

44.1kHz Actually two different sampling frequencies, 44.056kHz and 44.1kHz, are specified, but the two frequencies are almost similar (difference 0.1%), so that they can be considered the same.
These sampling frequencies were recommended by the Electronics Industries Association of Japan (EIA-J) and became popular with the CD.
Originally these frequencies where selected, because they can easily be synchronized with the horizontal frequency of video signals. This is important, if video-recorders are used for digital audio recordings. 44.056kHz is used for recording in NTSC-color-standard and 44.1kHz for NTSC-B/W-standard.
These sampling frequencies allows an audio band-width of up to 20kHz.

48kHz is used mainly for professional equipment. The audio band-width is also 20kHz. The advantage of this sampling frequency is , that it allows easy sampling frequency conversion to 32kHz and that $f_s/2$ is well beyond the audio range.
But the high sampling frequency produces a large data stream and thus requires expensive equipment.

If a certain sampling frequency is used, it must be made sure, that definitely no analogue signal above $f_s/2$ is applied to the sampling circuit. Any of such signal would produce aliasing. For this purpose the signal will be passed through a high quality low pass filter. According to its purpose it is called the
ANTI-ALIASING-FILTER.

The quality of these filters influence the quality of the A/D conversion to a large extent.

5. QUANTIZATION

After the sampling of the analogue signal, the analogue values of the samples must be converted to digital values. This process is called QUANTIZATION. It is done by the actual A/D CONVERTER.

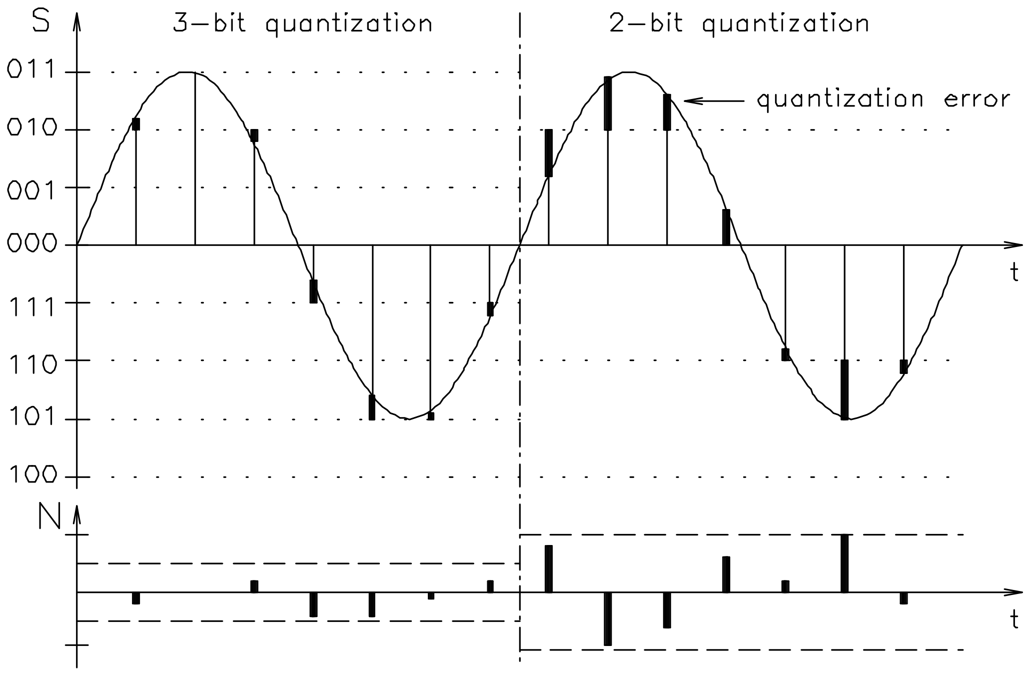


Fig. 1.5.1.
A sine wave is represented by a 2-bit and a 3-bit digital word. The lower diagrams show the difference between the actual analogue and digital signal. It represents the error of the digital signal.

The most important characteristic of such a converter is the number of bits, by which it represents the sample. The following fig. shows how the number of bits used will influence the precision of the conversion, using the simple example of a 2-bit and a 3-bit word:

It can clearly be seen by this example, that the 3-bit word, which allows a distinction of 8 different analogue values, approaches the original analogue signal much better than the 2-bit word.

The error signal, being the difference between the analogue and the digital signal, represents a new signal, added to the original analogue signal. So the error signal is just a NOISE SIGNAL. Due to its cause it is called the QUANTIZATION NOISE.

The quantization noise will become the smaller, the more bits are used for quantization. Obviously the error signal and thus the quantization noise is reduced to half (= -6dB) by every additional bit.

The UNWEIGHTED signal-to-noise ratio of the quantization process can be calculated by the formula:

$$S/N = n \cdot 6dB + 2dB$$

where: n is the number of bits
2dB is a factor of improvement of the noise, due to statistical cancellation of error signals.

Due to some other problems during quantization, the S/N-ratio will be somewhat lower than this theoretical value.

Modern A/D converts have resolution of up to 16-bit. This allows a S/N-ratio of up to 98dB. 16 bits allow 65536 steps of quantization. This means if the input signal has an amplitude of ±10V, one step is just 0.3mV. The circuit must be able to distinguish these small voltages.

6. BINARY SIGNAL CODES

There are different ways, how the analogue values can be represented in binary form. The agreement, which relates analogue values and binary values, is called the SIGNAL CODE.

Theoretically the binary digits could be arranged in random order,
e.g like this for a three bit quantization:

0 = 110	4 = 111
1 = 100	5 = 000
2 = 010	6 = 101
3 = 001	7 = 011

This code would fulfil the requirement, that for each decimal value there is just one binary code and vice versa. But the binary numbers are not at all meaningful and are not suitable for calculation.

A suitable code to represent the analogue signal values should fulfil the following requirements:

1. It must be able to distinguish between positive and negative values in order to be able to represent ac-signals.

2. It should allow simples procedures for addition of values and for multiplication of values.

In practice three different signal codes are used:

1. The Offset Code

The counting starts at the most negative analogue value with binary 0. It rises proportional with the analogue value to the highest possible binary number. At positive analogue values, the MSB of the binary number is 1, at negative values it is 0.

2. The One's Complement Code

The MSB of this code is 0 for positive analogue values and 1 for negative. The lower significant numbers are 0 at 0 analogue value. They count up towards positive values. The binary number for each negative analogue value is found by inverting each bit of the equivalent positive value. This code allows simple performance of multiplication.

3. The Two's Complement Code

The definition is similar to the one's complement, but the binary codes for the negative analogue values are found by subtracting the positive values from binary 0.
This code allows simple performance of addition and subtraction.
It is the code most commonly used for audio signals.

Quantizing Level	Offset – Binary Code	One´s Complement Code	Two´s Complement Code
+7	**1 1 1 1**	**0 1 1 1**	**0 1 1 1**
+6	**1 1 1 0**	**0 1 1 0**	**0 1 1 0**
+5	**1 1 0 1**	**0 1 0 1**	**0 1 0 1**
+4	**1 1 0 0**	**0 1 0 0**	**0 1 0 0**
+3	**1 0 1 1**	**0 0 1 1**	**0 0 1 1**
+2	**1 0 1 0**	**0 0 1 0**	**0 0 1 0**
+1	**1 0 0 1**	**0 0 0 1**	**0 0 0 1**
0	**1 0 0 0**	**0 0 0 0**	**0 0 0 0**
0	**(1 0 0 0)**	**1 1 1 1**	**(0 0 0 0)**
-1	**0 1 1 1**	**1 1 1 0**	**1 1 1 1**
-2	**0 1 1 0**	**1 1 0 1**	**1 1 1 0**
-3	**0 1 0 1**	**1 1 0 0**	**1 1 0 1**
-4	**0 1 0 0**	**1 0 1 1**	**1 1 0 0**
-5	**0 0 1 1**	**1 0 1 0**	**1 0 1 1**
-6	**0 0 1 0**	**1 0 0 1**	**1 0 1 0**
-7	**0 0 0 1**	**1 0 0 0**	**1 0 0 1**
-8	**0 0 0 0**		**1 0 0 0**

Fig. 1.6.1.
Example of 4bit representation of decimal values in different binary codes. The two's complement is the one commonly used for audio signals.

The conversion of one code to another can simply be performed by a ROM. The input code would represent the ROM-address, while the data in the memory cell would represent the output code. This method is fast, but it should be considered, that for the conversion of 16bit codes, 1Mbit of memory is required.

Therefore it might be more economical to convert codes by calculation.

To convert the one's complement code to the two's complement code, one could consider the following instructions:
- If the MSB = 0, leave the word as it is.
- If the MSB = 1, add 1 to the word.
These operations can be done in a simple digital circuit.

7. DYNAMIC RANGE OF DIGITAL SIGNALS

The dynamic range of a signal describes the range between the smallest signal that can be distinguished from the noise level and the highest signal the system can handle.

The smallest signal in a digital system is one bit. Any smaller signal will disappear in the quantization noise. The largest signal the system can represent is given by the highest binary number of the code.

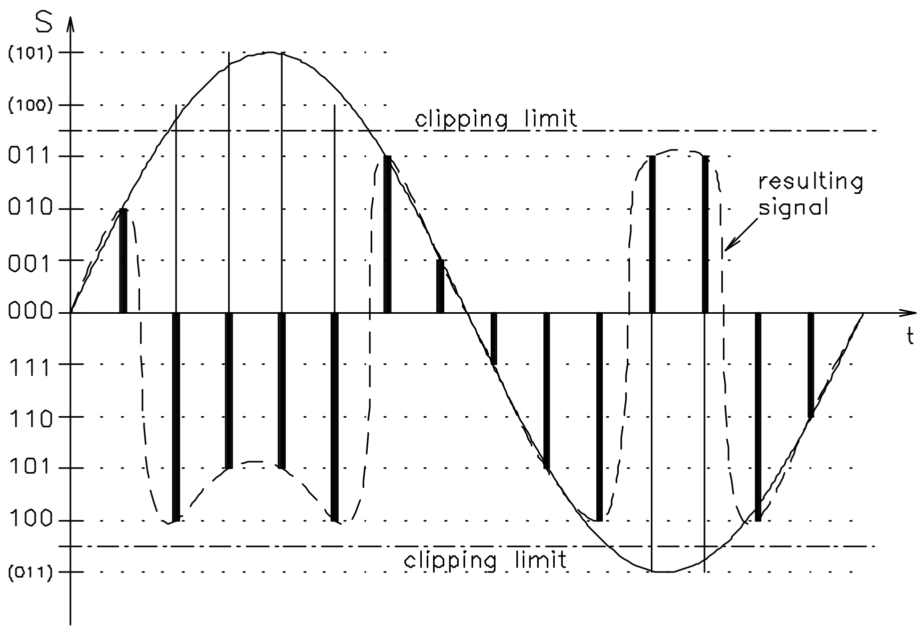


Fig. 1.7.1.
Example of a 3-bit quantization.
The quantization of the analogue signal exceeds the range of the binary numbers. When the converter reaches its highest value, it continues at the most negative values. This produces very strong distortions of the signal.

In case of OVER-MODULATION, the analogue signal results in a binary output which exceeds the number range of the code, the converter would produce an OVERFLOW and the output data are wrong.

Modern A/D-converters avoid such distortions. They produce constantly the highest binary value as long as the overflow occurs. This results in an output signal, which is equivalent to the well-known CLIPPING of analogue systems.

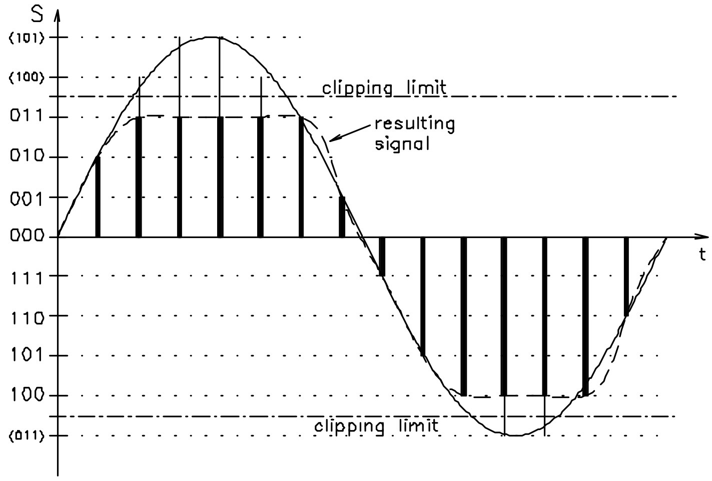


Fig. 1.7.2.
The same situation as in Fig. 1.7.1., but the converter produces constantly the highest binary value, as long as the overflow occurs. It can be seen, that the result is not that severe.

Nevertheless in digital systems over-modulation is a more serious problem than in analogue systems. In analogue systems (amplifiers, tape recorders etc.) the distortions rise slowly, when the maximum allowed level is exceeded (graceful degradation). In digital systems the change from no distortions to strong distortions happens suddenly. Furthermore the clipped signal now contains harmonics, which may contain frequencies beyond the Nyquist frequency. These may again produce aliasing. Therefore it is essential that over-modulation is prevented in digital audio.

For this reason the normally allowed maximum level is put somewhat below the clipping limit. This difference is then available as reserve for accidental over-modulation. This reserve is known as HEAD ROOM.

As the head room is not available for normal signal levels, it means a reduction of the dynamic. A compromise has to be found between a wide dynamic range and sufficient head room.

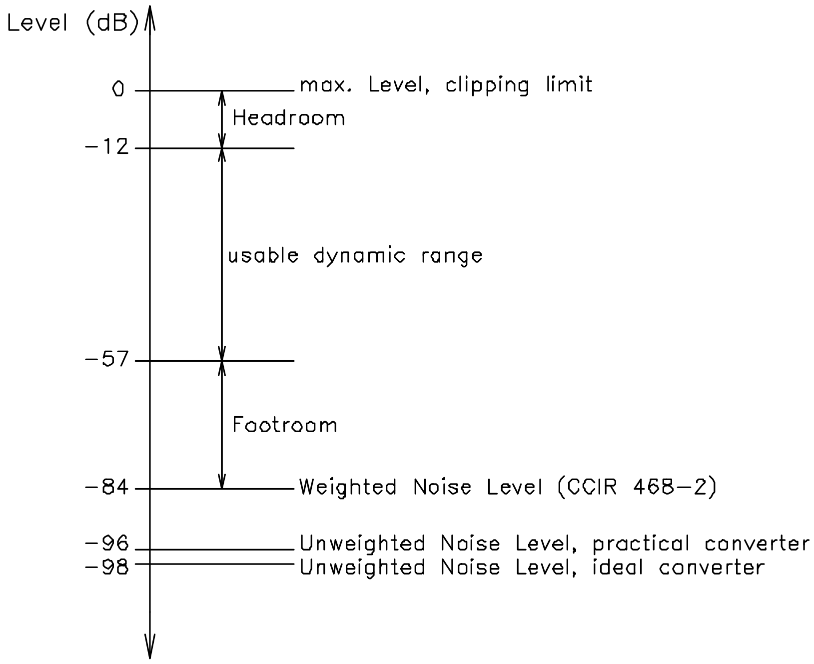


Fig. 1.7.3.
The whole level range of a 16-bit quantized signal.
The unweighted quantization noise lies 98db below the maximum level. The weighted noise (CCIR 468) is 12db higher, leaving 84dB of weighted system dynamic. The head room may be selected with 12dB, leaving a weighted S/N ratio of 72dB.

It can be seen, that this dynamic range is far beyond any practical requirement.

8. COMPARING ANALOGUE AND DIGITAL SIGNALS

If the digital sound processing (DSP) requires an increased expenditure in equipment, then there must be some advantage about it. Let us compare the sound quality criteria of analogue and digital sound processing.

For analogue audio equipment, we have certain parameters, which are used to judge the quality of the equipment. Here are the major ones and the typical values for professional broadcasting equipment:

Parameter	amplifier	tape recorders	transmission
S/N ratio	100dB	70dB	60dB
Harmonic distortions	0.05%	0.5%	0.5%
Frequency response (20Hz – 20kHz)	±0.5dB	±1dB	±1dB
Wow and flutter	- / -	0.05%	- / -

It is a characteristic of the analogue signal processing, that with each step of processing, more unwanted signals (noise, harmonics, etc.) are added to the original signal.
Signal and noise can not be separated any more.

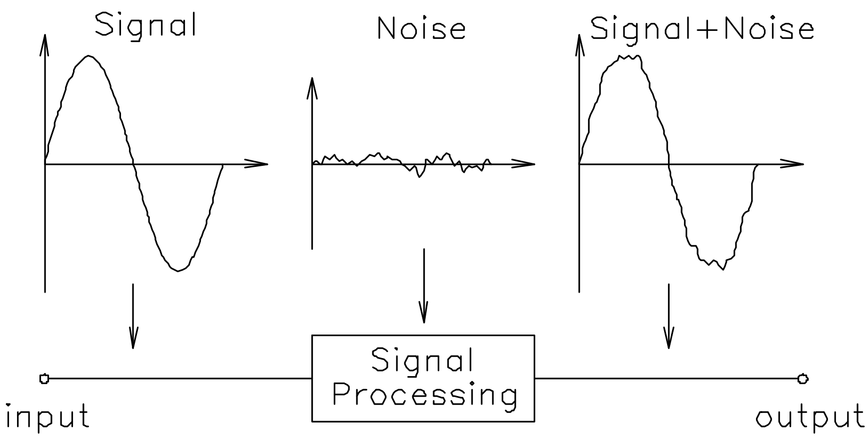


Fig. 1.8.1.
When audio signals are processed, unwanted signals are added irreversible to the signal. Each further process adds more of these signals.

The DSP does not produce any of the above mentioned changes to the original signals at all. (theoretically, the exceptions will be discussed later!)

But the original analogue signal must be converted to a digital one and must be converted back to analogue finally. These conversions are producing the only noise and harmonic distortions in the system

So it must be the aim of the system to do as few conversions as possible, and to convert the analogue signal as soon as possible and to convert it back as late as possible.

The following table shows the quality parameter of professional digital audio equipment:

Parameter	A / D – D / A conversion	tape recorders	transmission
S/N ratio	90 dB	- / -	- / -
Harmonic distortions	0.05%	- / -	- / -
Frequency responce (20Hz – 20kHz)	±0.5dB	- / -	- / -
Wow and flutter	0	- / -	- / -

This table shows: the digital signal can be recorded, transmitted (via cable, optical fibre, satellite) and processed as much as necessary, it will not effect the quality. This is often considered the the biggest advantage of DSP.

It is not so, that DSP allows better sound quality than analogue equipment.
Modern analogue equipment provides sound qualities, which is anyhow beyond human recognition. There would be no need to improve that. But DSP allows PROCESSING, STORAGE and TRANSMISSION of signals without any MORE losses in sound quality.

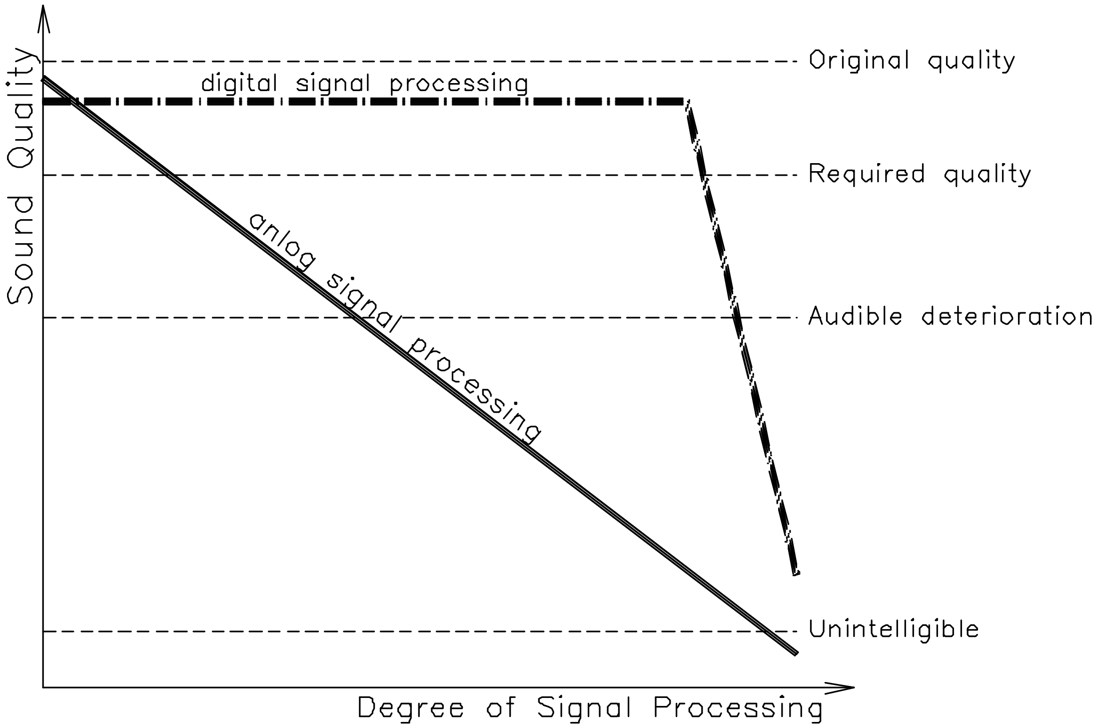


Fig. 1.8.2.
The quality of analogue signal gets the poorer the more the signal is processed and the farer it is transmitted.
Digital signals are not effected by processing or transmission, as long as a certain limit is not exceeded.

The costs for DSP-equipment were decaying during the last years. Today consumer digital equipment is cheaper than professional analogue equipment. Due to the high sound quality of digital consumer equipment it is partly replacing professional products.

With the introduction of computerized broadcasting another step towards cheaper equipment for professional use can be expected.

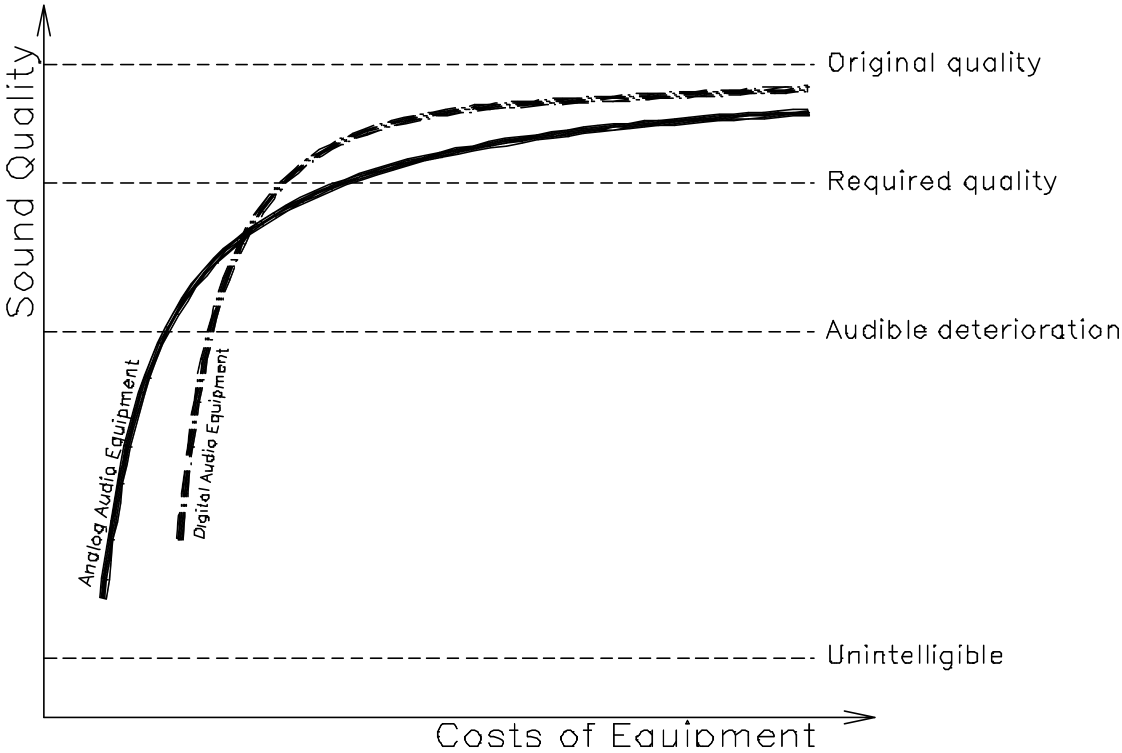


Fig. 1.8.3.
The diagram shows roughly the relation between achievable sound quality and the price of analogue and digital equipment:
For analogue equipment the costs increase somehow with the sound quality.
Digital equipment requires higher minimum costs, but offer high sound quality, even for simple solutions.
Digital equipment has a tendency of becoming cheaper.